Abstract

This report explains how a special compound of twelve cubes can be derived. The compound has the same symmetry as one cube and it belongs to the symmetry group $12|S_4 \times I/D_1 \times I$. This compound has however a property that does not apply to the other compounds in this group and that is that this compound consists of three Bakos compounds ($4|S_4 \times I/D_3 \times I$). This report defines all the cubes and derives the building templates.

1 Three Bakos Compounds

For all compounds that belong to the symmetry group symmetry group

$$12|S_4 \times I/D_1 \times I|\mu$$

holds that they can be divided into six subcompounds of $2|D_2 \times I/D_1 \times I$ in two ways, or into six subcompounds $2|D_4 \times I/D_2 \times I$, However for $\mu = \arccos \frac{7}{9} = \arctan \frac{4\sqrt{2}}{7}$ the compound consists of $3 \times 4|S_4 \times I/D_3 \times I$, i.e. three Bakos compounds as well. This compound can be obtained by replacing each cube in the classic compound of three cubes by a Bakos compound, according to the symmetry. This can be seen as multiplying Bakos' compound with the classic compound of three cubes. (Note that this multiplication is not commutative.)

The compound can be derived from one cube by some matrix multiplications. If the original cube is defined by the combinations of unit coordinates then the following matrices will transfer the original cube into the compound of 12 cubes.

$$\begin{bmatrix} 0 & -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ \frac{2}{3}\sqrt{2} & \frac{1}{6}\sqrt{2} & \frac{1}{6}\sqrt{2} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 0 & -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{2}{3}\sqrt{2} & \frac{1}{6}\sqrt{2} & -\frac{1}{6}\sqrt{2} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3}\sqrt{2} & -\frac{1}{6}\sqrt{2} & \frac{1}{6}\sqrt{2} \\ 0 & \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ -\frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} \frac{2}{3}\sqrt{2} & -\frac{1}{6}\sqrt{2} & -\frac{1}{6}\sqrt{2} \\ 0 & \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} & 0 \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6}\sqrt{2} & \frac{1}{6}\sqrt{2} & \frac{2}{3}\sqrt{2} \end{bmatrix} \begin{bmatrix} \frac{1}{6}\sqrt{2} & \frac{1}{6}\sqrt{2} & -\frac{2}{3}\sqrt{2} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & 0 \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{6}\sqrt{2} & -\frac{1}{6}\sqrt{2} & \frac{2}{3}\sqrt{2} \end{bmatrix} \begin{bmatrix} \frac{1}{6}\sqrt{2} & -\frac{1}{6}\sqrt{2} & -\frac{2}{3}\sqrt{2} \\ -\frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{2}\sqrt{2} & 0 & -\frac{1}{2}\sqrt{2} \\ \frac{1}{6}\sqrt{2} & \frac{2}{3}\sqrt{2} & \frac{1}{6}\sqrt{2} \end{bmatrix} \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \frac{1}{6}\sqrt{2} & \frac{2}{3}\sqrt{2} & -\frac{1}{6}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & 0 & \frac{1}{2}\sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{1}{6}\sqrt{2} & \frac{2}{3}\sqrt{2} & -\frac{1}{6}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & 0 & \frac{1}{2}\sqrt{2} \end{bmatrix} \begin{bmatrix} \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ -\frac{1}{2}\sqrt{2} & 0 & -\frac{1}{2}\sqrt{2} \\ -\frac{1}{6}\sqrt{2} & \frac{2}{3}\sqrt{2} & \frac{1}{6}\sqrt{2} \end{bmatrix}$$

The vertex matrices that are obtained by these multiplication are as follows:

$$\begin{bmatrix} -\sqrt{2} & \frac{2}{3}\sqrt{2} & -\frac{1}{3} \\ -\sqrt{2} & -\frac{2}{3}\sqrt{2} & \frac{1}{3} \\ 0 & -\sqrt{2} & -1 \\ 0 & \frac{1}{3}\sqrt{2} & -\frac{5}{3} \\ 0 & \sqrt{2} & 1 \\ 0 & -\frac{1}{3}\sqrt{2} & \frac{5}{3} \\ \sqrt{2} & -\frac{2}{3}\sqrt{2} & \frac{1}{3} \\ \sqrt{2} & \frac{2}{3}\sqrt{2} & -\frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 & \sqrt{2} & -1 \\ 0 & -\frac{1}{3}\sqrt{2} & -\frac{5}{3} \\ \sqrt{2} & -\frac{2}{3}\sqrt{2} & -\frac{1}{3} \\ \sqrt{2} & \frac{2}{3}\sqrt{2} & \frac{1}{3} \\ -\sqrt{2} & \frac{2}{3}\sqrt{2} & \frac{1}{3} \\ -\sqrt{2} & -\frac{2}{3}\sqrt{2} & -\frac{1}{3} \\ 0 & -\sqrt{2} & 1 \\ 0 & \frac{1}{3}\sqrt{2} & \frac{5}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{3}\sqrt{2} & 0 & -\frac{5}{3} \\ -\sqrt{2} & 0 & -1 \\ -\frac{2}{3}\sqrt{2} & -\sqrt{2} & \frac{1}{3} \\ \frac{2}{3}\sqrt{2} & -\sqrt{2} & -\frac{1}{3} \\ \frac{2}{3}\sqrt{2} & \sqrt{2} & -\frac{1}{3} \\ -\frac{2}{3}\sqrt{2} & \sqrt{2} & \frac{1}{3} \\ -\frac{1}{3}\sqrt{2} & 0 & \frac{5}{3} \\ \sqrt{2} & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{2}{3}\sqrt{2} & \sqrt{2} & \frac{1}{3} \\ -\frac{2}{3}\sqrt{2} & \sqrt{2} & -\frac{1}{3} \\ -\frac{1}{3}\sqrt{2} & 0 & -\frac{5}{3} \\ \sqrt{2} & 0 & -1 \\ \frac{1}{3}\sqrt{2} & 0 & \frac{5}{3} \\ -\sqrt{2} & 0 & 1 \\ -\frac{2}{3}\sqrt{2} & -\sqrt{2} & -\frac{1}{3} \\ \frac{2}{3}\sqrt{2} & -\sqrt{2} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0 & \frac{5}{3} & -\frac{1}{3}\sqrt{2} \\ -\sqrt{2} & \frac{1}{3} & -\frac{2}{3}\sqrt{2} \\ 0 & -1 & -\sqrt{2} \\ \sqrt{2} & \frac{1}{3} & -\frac{2}{3}\sqrt{2} \\ 0 & 1 & \sqrt{2} \\ -\sqrt{2} & -\frac{1}{3} & \frac{2}{3}\sqrt{2} \\ 0 & -\frac{5}{3} & \frac{1}{3}\sqrt{2} \\ \sqrt{2} & -\frac{1}{3} & \frac{2}{3}\sqrt{2} \end{bmatrix} \begin{bmatrix} \sqrt{2} & 1 & 0 \\ \frac{2}{3}\sqrt{2} & -\frac{1}{3} & -\sqrt{2} \\ \frac{1}{3}\sqrt{2} & -\frac{5}{3} & 0 \\ \frac{2}{3}\sqrt{2} & -\frac{1}{3} & \sqrt{2} \\ -\frac{1}{3}\sqrt{2} & \frac{5}{3} & 0 \\ -\frac{2}{3}\sqrt{2} & \frac{1}{3} & -\sqrt{2} \\ -\sqrt{2} & -1 & 0 \\ -\frac{2}{3}\sqrt{2} & \frac{1}{3} & \sqrt{2} \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{2} & -\frac{1}{3} & -\frac{2}{3}\sqrt{2} \\ 0 & 1 & -\sqrt{2} \\ -\sqrt{2} & -\frac{1}{3} & -\frac{2}{3}\sqrt{2} \\ 0 & -\frac{5}{3} & -\frac{1}{3}\sqrt{2} \\ \sqrt{2} & \frac{1}{3} & \frac{2}{3}\sqrt{2} \\ 0 & \frac{5}{3} & \frac{1}{3}\sqrt{2} \\ -\sqrt{2} & \frac{1}{3} & \frac{2}{3}\sqrt{2} \\ 0 & -1 & \sqrt{2} \end{bmatrix} \begin{bmatrix} \frac{2}{3}\sqrt{2} & \frac{1}{3} & \sqrt{2} \\ \frac{1}{3}\sqrt{2} & \frac{5}{3} & 0 \\ \frac{2}{3}\sqrt{2} & \frac{1}{3} & -\sqrt{2} \\ \sqrt{2} & -1 & 0 \\ -\frac{2}{3}\sqrt{2} & -\frac{1}{3} & \sqrt{2} \\ -\sqrt{2} & 1 & 0 \\ -\frac{2}{3}\sqrt{2} & -\frac{1}{3} & -\sqrt{2} \\ -\frac{1}{3}\sqrt{2} & -\frac{5}{3} & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{3} & \sqrt{2} & \frac{2}{3}\sqrt{2} \\ -\frac{5}{3} & 0 & \frac{1}{3}\sqrt{2} \\ -1 & 0 & -\sqrt{2} \\ \frac{1}{3} & \sqrt{2} & -\frac{2}{3}\sqrt{2} \\ 1 & 0 & \sqrt{2} \\ -\frac{1}{3} & -\sqrt{2} & \frac{2}{3}\sqrt{2} \\ \frac{1}{3} & -\sqrt{2} & -\frac{2}{3}\sqrt{2} \\ \frac{5}{3} & 0 & -\frac{1}{3}\sqrt{2} \end{bmatrix}$$

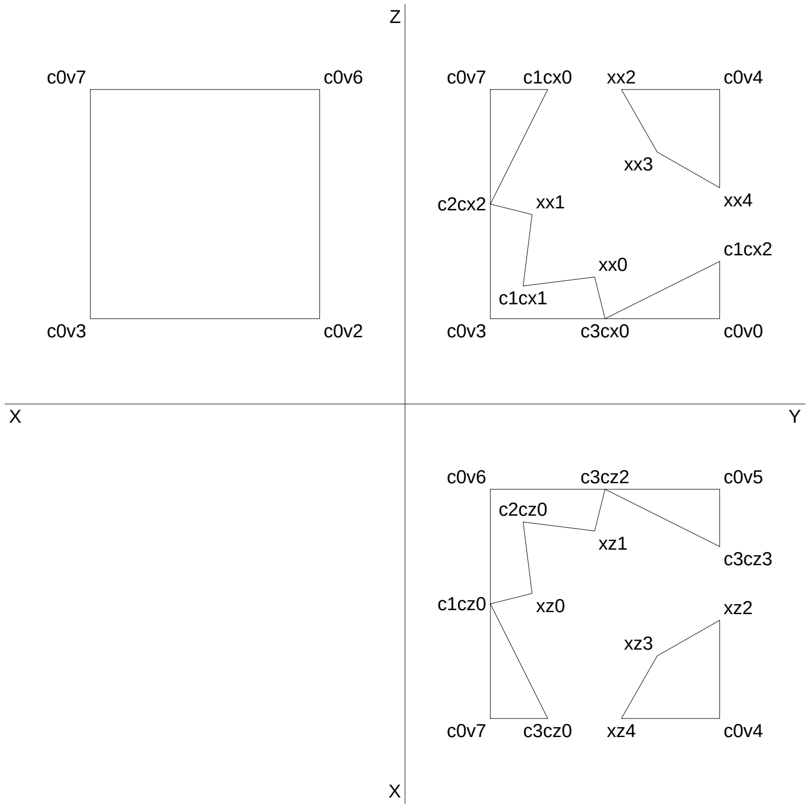

Figure 1: Intersections between The Base Cube and The First 3 Cubes

$$
\begin{bmatrix}
1 & \sqrt{2} & 0 \\
-\frac{1}{3} & \frac{2}{3}\sqrt{2} & -\sqrt{2} \\
\frac{1}{3} & -\frac{2}{3}\sqrt{2} & -\sqrt{2} \\
\frac{5}{3} & -\frac{1}{3}\sqrt{2} & 0 \\
-\frac{1}{3} & \frac{2}{3}\sqrt{2} & \sqrt{2} \\
-\frac{5}{3} & \frac{1}{3}\sqrt{2} & 0 \\
-1 & -\sqrt{2} & 0 \\
\frac{1}{3} & -\frac{2}{3}\sqrt{2} & \sqrt{2}
\end{bmatrix}
\begin{bmatrix}
\frac{1}{3} & \frac{2}{3}\sqrt{2} & -\sqrt{2} \\
-1 & \sqrt{2} & 0 \\
-\frac{5}{3} & -\frac{1}{3}\sqrt{2} & 0 \\
-\frac{1}{3} & -\frac{2}{3}\sqrt{2} & -\sqrt{2} \\
\frac{5}{3} & \frac{1}{3}\sqrt{2} & 0 \\
\frac{1}{3} & \frac{2}{3}\sqrt{2} & \sqrt{2} \\
-\frac{1}{3} & -\frac{2}{3}\sqrt{2} & \sqrt{2} \\
1 & -\sqrt{2} & 0
\end{bmatrix}
\begin{bmatrix}
\frac{5}{3} & 0 & \frac{1}{3}\sqrt{2} \\
\frac{1}{3} & \sqrt{2} & \frac{2}{3}\sqrt{2} \\
-\frac{1}{3} & \sqrt{2} & -\frac{2}{3}\sqrt{2} \\
1 & 0 & -\sqrt{2} \\
\frac{1}{3} & -\sqrt{2} & \frac{2}{3}\sqrt{2} \\
-1 & 0 & \sqrt{2} \\
-\frac{5}{3} & 0 & -\frac{1}{3}\sqrt{2} \\
-\frac{1}{3} & -\sqrt{2} & -\frac{2}{3}\sqrt{2}
\end{bmatrix}
$$

To calculate the intersections between the first cube and all the others it is easier to translate all the cubes in such a way that the first cube becomes a unit cube again. This can done by multiplying all vertices with the inverse matrix that was used for the first cube. This first cube becomes the base cube now. Only two orthogonal projections are needed for this compound, the third will be the same as one of the others.

The first four cubes form a Bakos compound, so for the intersections between the base cube and the first three cubes the intersections of the Bakos compound can be used. This is shown in Figure 1. The values are as follows:

$$\begin{array}{lcl}
c1cx0 & = & (\ -\frac{1}{2}, \ 1\ ) \\
c1cx1 & = & (\ -\frac{5}{7}, \ -\frac{5}{7}\ ) \\
c1cx2 & = & (\ 1, \ -\frac{1}{2}\ ) \\
c2cx2 & = & (\ -1, \ 0\ ) \\
c3cx0 & = & (\ 0, \ -1\ ) \\
xx0 & = & (\ -\frac{1}{11}, -\frac{7}{11}\ ) \\
xx1 & = & (\ -\frac{7}{11}, -\frac{1}{11}\ ) \\
xx2 & = & (\ \frac{1}{7}, \ 1\ ) \\
xx3 & = & (\ \frac{5}{11}, \ \frac{5}{11}\ ) \\
xx4 & = & (\ 1, \ \frac{1}{7}\ ) \\
c1cz0 & = & (\ 0, \ -1\ ) \\
c2cz0 & = & (\ -\frac{5}{7}, \ -\frac{5}{7}\ ) \\
c3cz0 & = & (\ 1, \ -\frac{1}{2}\ ) \\
c3cz2 & = & (\ -1, \ 0\ ) \\
c3cz3 & = & (\ -\frac{1}{2}, \ 1\ ) \\
xz0 & = & (\ -\frac{1}{11}, -\frac{7}{11}\ ) \\
xz1 & = & (\ -\frac{7}{11}, -\frac{1}{11}\ ) \\
xz2 & = & (\ \frac{1}{7}, \ 1\ ) \\
xz3 & = & (\ \frac{5}{11}, \ \frac{5}{11}\ ) \\
xz4 & = & (\ 1, \ \frac{1}{7}\ )
\end{array}$$

Figure 2 shows the fifth cube relative to the base cube. With this picture one can calculate the intersections between these two cubes. Figure 3 shows these intersections between the base cube and the fifth cube. The labels have the following values:

$$\begin{array}{lcl}
c_5cx_0 & = & (\ \frac{15-9\sqrt{2}}{7}, \ -1\ ) \\
c_5cx_1 & = & (\ -1, \ \frac{15-9\sqrt{2}}{7}\ ) \\
c_5cx_2 & = & (\ \frac{9\sqrt{2}-7}{8}, \ \frac{9\sqrt{2}-7}{8}\ ) \\
c_5cz_0 & = & (\ \frac{9\sqrt{2}-8}{7}, \ 1\ ) \\
c_5cz_1 & = & (\ \frac{18\sqrt{2}-23}{7}, \frac{207\sqrt{2}-233}{49}\ ) \\
c_5cz_2 & = & (\ 0, \ -1\ ) \\
c_5cz_3 & = & (\ -1, \ \frac{9\sqrt{2}-15}{7}\ )
\end{array}$$

Figure 4 shows the sixth cube relative to the base cube. With this picture one can calculate the intersections between these two cubes. Figure 5 shows these intersections

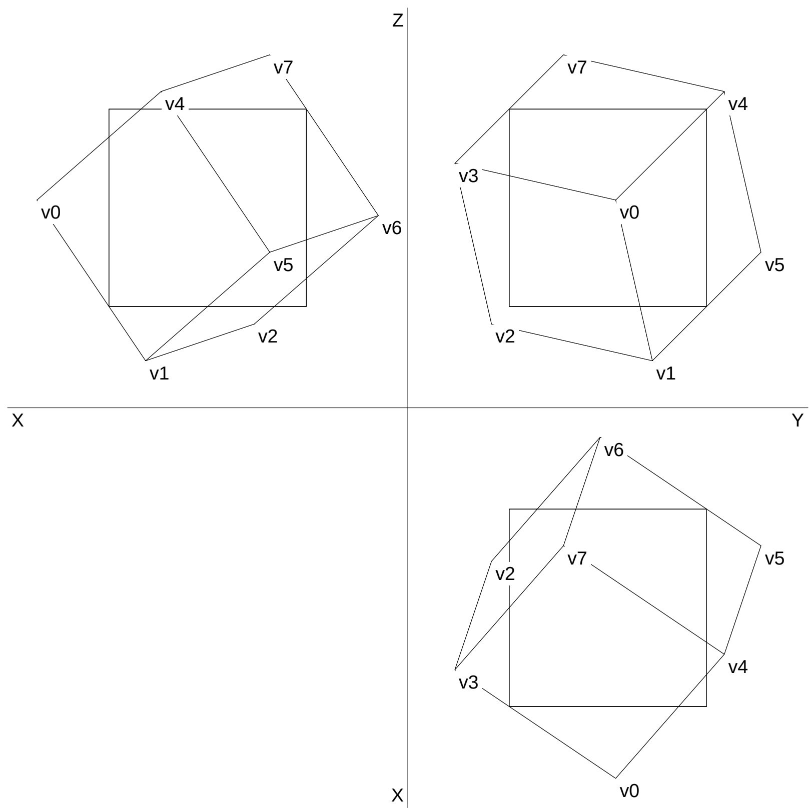

Figure 2: The Fifth Cube and Base Cube

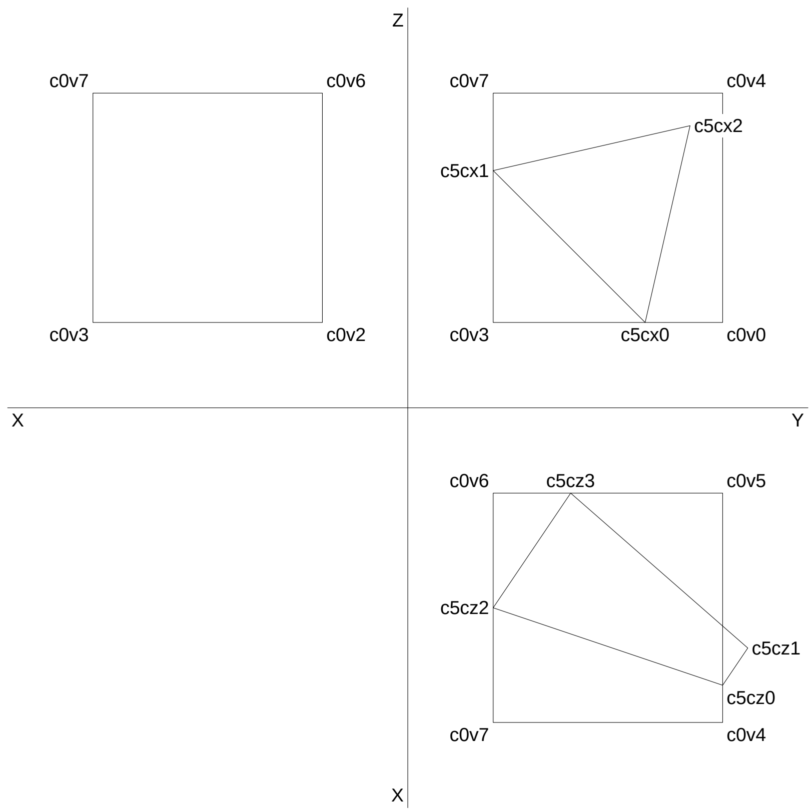

Figure 3: Intersections between The Base Cube and The Fifth Cube

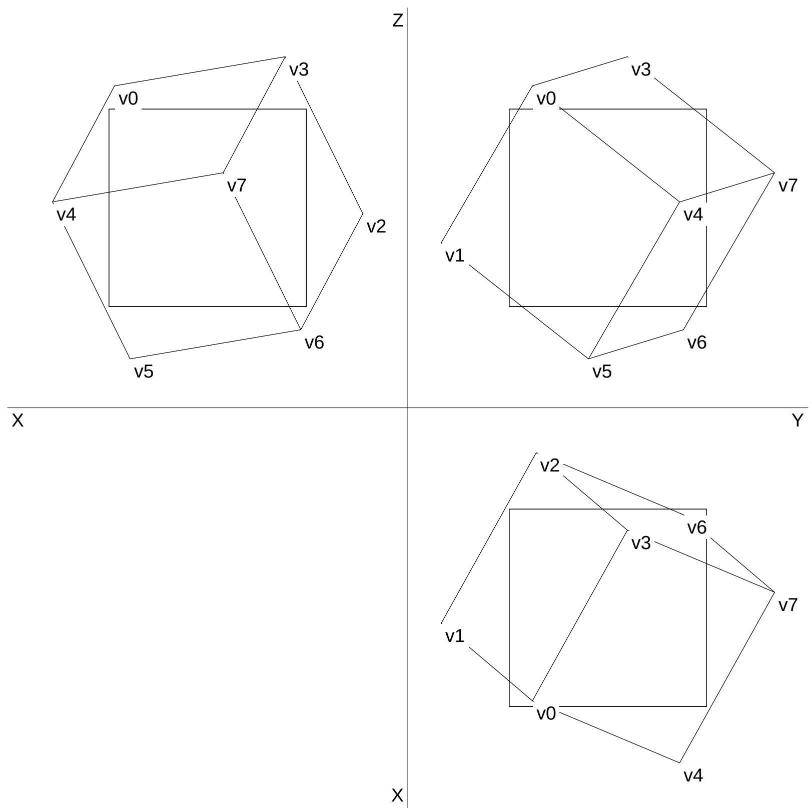

Figure 4: The Sixth Cube and Base Cube

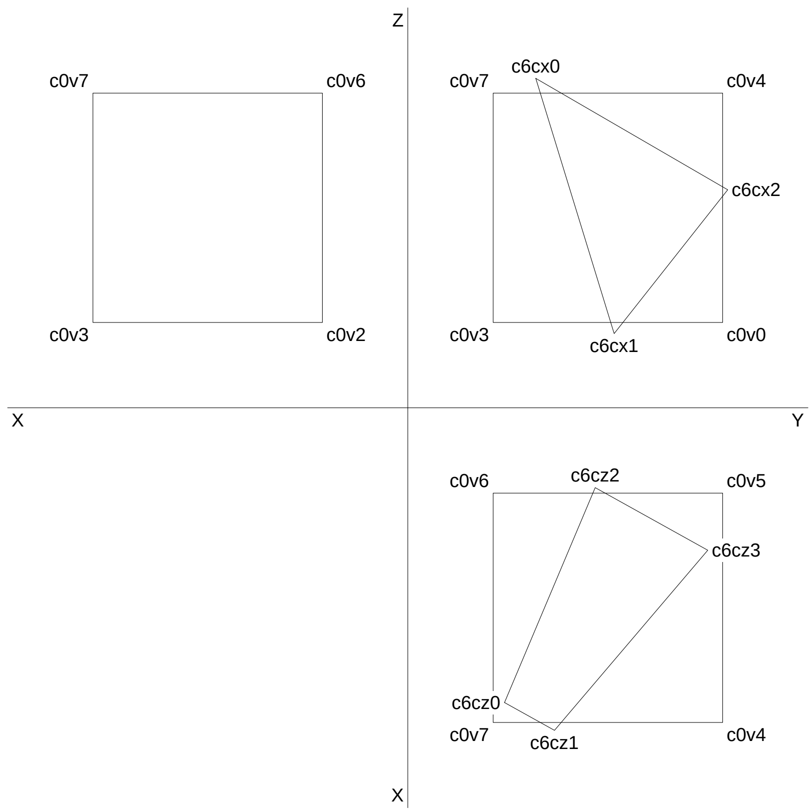

Figure 5: Intersections between The Base Cube and The Sixth Cube

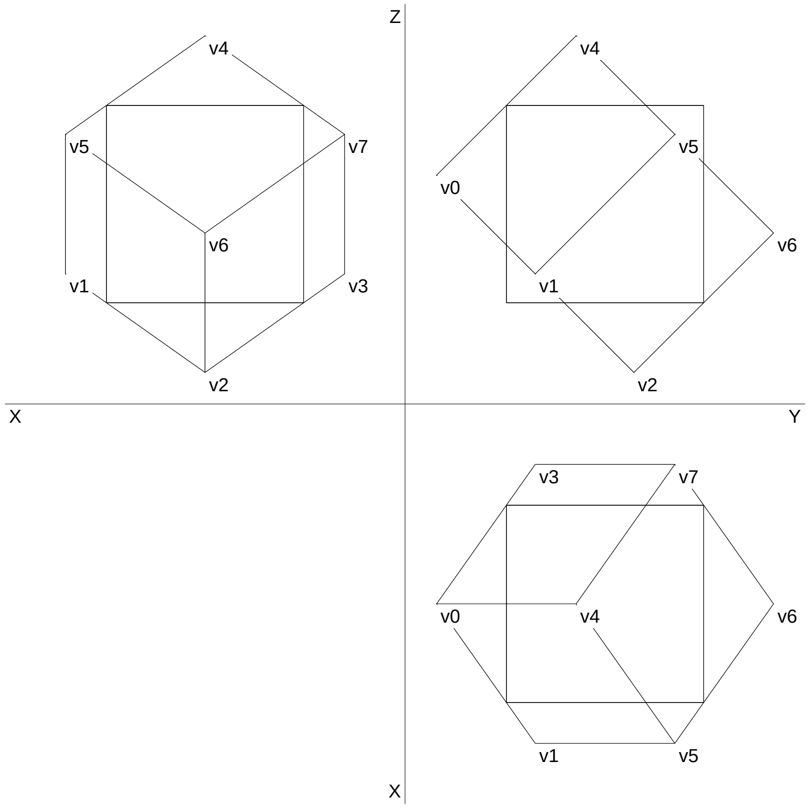

Figure 6: The Seventh Cube and Base Cube

between the base cube and the sixth cube. The labels have the following values:

$$
\begin{array}{ll}
c_6cx_0 = (& \frac{6\sqrt{2}-11}{4}, \ \frac{-6\sqrt{2}+13}{4} \ ) \\
c_6cx_1 = (& \frac{-9\sqrt{2}+13}{5}, \ \frac{-6\sqrt{2}+3}{5} \ ) \\
c_6cx_2 = (& \frac{6\sqrt{2}+3}{11}, \ \frac{9\sqrt{2}-11}{11} \ ) \\
c_6cz_0 = (& \frac{66\sqrt{2}+5}{119}, \ \frac{-3\sqrt{2}-103}{119} \ ) \\
c_6cz_1 = (& \frac{48\sqrt{2}+8}{71}, \ \frac{24\sqrt{2}-67}{71} \ ) \\
c_6cz_2 = (& \frac{-120\sqrt{2}+45}{119}, \ \frac{-27\sqrt{2}+25}{119} \ ) \\
c_6cz_3 = (& \frac{-45\sqrt{2}+28}{71}, \ \frac{84\sqrt{2}-57}{71} \ )
\end{array}
$$

Figure 6 shows the seventh cube relative to the base cube. With this picture one can calculate the intersections between these two cubes. Figure 7 shows these intersections between the base cube and the seventh cube. From these one can recognise the intersections of a $2|D_4 \times I/D_2 \times I$. The labels have the following values:

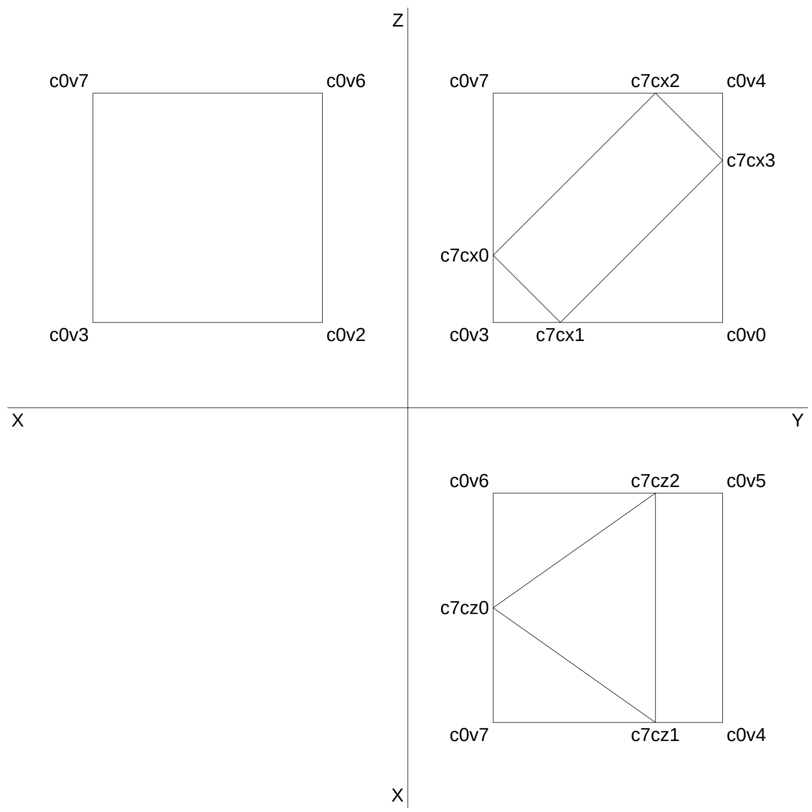

Figure 7: Intersections between The Base Cube and The Seventh Cube

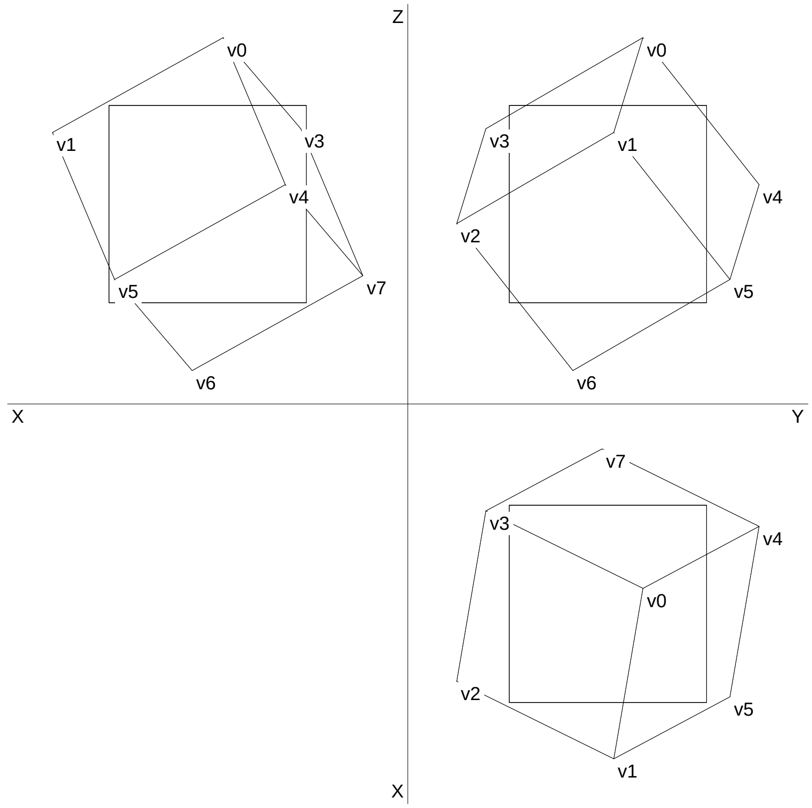

Figure 8: The Eighth Cube and Base Cube

$$
\begin{aligned}
c_7cx_0 &= (\quad -1, -\sqrt{2}+1 \,) \\
c_7cx_1 &= (-\sqrt{2}+1, \quad -1 \,) \\
c_7cx_2 &= (\sqrt{2}-1, \quad 1 \,) \\
c_7cx_3 &= (\quad 1, \quad \sqrt{2}-1 \,) \\
c_7cz_0 &= (\quad 0, \quad -1 \,) \\
c_7cz_1 &= (\quad 1, \quad \sqrt{2}-1 \,) \\
c_7cz_2 &= (\quad -1, \quad \sqrt{2}-1 \,)
\end{aligned}
$$

Figure 8 shows the eighth cube relative to the base cube. With this picture one can calculate the intersections between these two cubes. Figure 9 shows these intersections between the base cube and the eigth cube. The labels have the following values:

$$
\begin{aligned}
c_8cx_0 &= (\quad \tfrac{9\sqrt{2}-11}{11}, \quad \tfrac{6\sqrt{2}+3}{11} \,) \\
c_8cx_1 &= (\quad \tfrac{-6\sqrt{2}+3}{5}, \quad \tfrac{-9\sqrt{2}+13}{5} \,)
\end{aligned}
$$

$$\begin{aligned}
c_8cx_2 &= (\tfrac{-6\sqrt{2}+13}{4}, & \tfrac{6\sqrt{2}-11}{4}) \\
c_8cz_0 &= (\tfrac{72\sqrt{2}-77}{23}, & \tfrac{-21\sqrt{2}+33}{23}) \\
c_8cz_1 &= (\tfrac{-66\sqrt{2}+5}{119}, & \tfrac{3\sqrt{2}-103}{119}) \\
c_8cz_2 &= (\tfrac{3\sqrt{2}-36}{71}, & \tfrac{108\sqrt{2}-89}{71})
\end{aligned}$$

Figure 10 shows the nineth cube relative to the base cube. With this picture one can calculate the intersections between these two cubes. Figure 11 shows these intersections between the base cube and the nineth cube. The labels have the following values:

$$\begin{aligned}
c_9cx_0 &= (\tfrac{54\sqrt{2}+9}{71}, & \tfrac{9\sqrt{2}+37}{71}) \\
c_9cx_1 &= (\tfrac{39\sqrt{2}+29}{71}, & \tfrac{42\sqrt{2}-7}{71}) \\
c_9cx_2 &= (\tfrac{-39\sqrt{2}+29}{71}, & \tfrac{-42\sqrt{2}-7}{71}) \\
c_9cx_3 &= (\tfrac{-54\sqrt{2}+9}{71}, & \tfrac{-9\sqrt{2}+37}{71}) \\
c_9cz_0 &= (\tfrac{-78\sqrt{2}+179}{119}, & \tfrac{69\sqrt{2}+11}{119}) \\
c_9cz_1 &= (\tfrac{120\sqrt{2}-283}{119}, & \tfrac{141\sqrt{2}-157}{119}) \\
c_9cz_2 &= (\tfrac{3\sqrt{2}-4}{5}, & \tfrac{-12\sqrt{2}+11}{5})
\end{aligned}$$

Figure 12 shows the tenth cube relative to the base cube. With this picture one can calculate the intersections between these two cubes. Figure 13 shows these intersections between the base cube and the tenth cube. The labels have the following values:

$$\begin{aligned}
c_{10}cx_0 &= (\tfrac{-9\sqrt{2}+4}{16}, & \tfrac{9\sqrt{2}+4}{16}) \\
c_{10}cx_1 &= (-9\sqrt{2}+13, & 1) \\
c_{10}cx_2 &= (\tfrac{-9\sqrt{2}+2}{16}, & \tfrac{-15\sqrt{2}+6}{16}) \\
c_{10}cx_3 &= (15\sqrt{2}-21, & -1) \\
c_{10}cz_0 &= (\tfrac{72\sqrt{2}+281}{287}, & \tfrac{-135\sqrt{2}+83}{287}) \\
c_{10}cz_1 &= (1, & -9\sqrt{2}+13) \\
c_{10}cz_2 &= (\tfrac{120\sqrt{2}-297}{287}, & \tfrac{-225\sqrt{2}-53}{287}) \\
c_{10}cz_3 &= (-1, & -15\sqrt{2}+21)
\end{aligned}$$

Figure 14 shows the eleventh cube relative to the base cube. With this picture one can calculate the intersections between these two cubes. Figure 15 shows these intersections between the base cube and the eleventh cube. The labels have the following values:

$$c_{11}cx_0 = (\tfrac{-15\sqrt{2}+6}{16}, \quad \tfrac{-9\sqrt{2}+2}{16})$$

$$
\begin{aligned}
c_{11}cx_1 &= (-1,\ 15\sqrt{2}-21)\\
c_{11}cx_2 &= (\tfrac{9\sqrt{2}+4}{16},\ \tfrac{-9\sqrt{2}+4}{16})\\
c_{11}cx_3 &= (1,\ -9\sqrt{2}+13)\\
c_{11}cz_0 &= (\tfrac{18\sqrt{2}+145}{287},\ \tfrac{-249\sqrt{2}+51}{287})\\
c_{11}cz_1 &= (48\sqrt{2}-68,\ 24\sqrt{2}-35)\\
c_{11}cz_2 &= (\tfrac{-72\sqrt{2}+281}{287},\ \tfrac{135\sqrt{2}+83}{287})\\
c_{11}cz_3 &= (45\sqrt{2}-64,\ 24\sqrt{2}-33)
\end{aligned}
$$

Figure 16 shows the twelfth cube relative to the base cube. With this picture one can calculate the intersections between these two cubes. Figure 17 shows these intersections between the base cube and the twelfth cube. The labels have the following values:

$$
\begin{aligned}
c_{12}cx_0 &= (\tfrac{9\sqrt{2}+37}{71},\ \tfrac{54\sqrt{2}+9}{71})\\
c_{12}cx_1 &= (\tfrac{42\sqrt{2}-7}{71},\ \tfrac{39\sqrt{2}+29}{71})\\
c_{12}cx_2 &= (\tfrac{-42\sqrt{2}-7}{71},\ \tfrac{-39\sqrt{2}+29}{71})\\
c_{12}cx_3 &= (\tfrac{-9\sqrt{2}+37}{71},\ \tfrac{-54\sqrt{2}+9}{71})\\
c_{12}cz_0 &= (\tfrac{3\sqrt{2}-4}{11},\ \tfrac{-12\sqrt{2}+5}{11})\\
c_{12}cz_1 &= (\tfrac{72\sqrt{2}-123}{23},\ \tfrac{99\sqrt{2}-149}{23})\\
c_{12}cz_2 &= (\tfrac{12}{11},\ \tfrac{7}{11})\\
c_{12}cz_3 &= (\tfrac{42\sqrt{2}-43}{23},\ \tfrac{75\sqrt{2}-85}{23})
\end{aligned}
$$

Figure 18 shows the X-template for building the compound of 12 cubes, constisting of 3 Bakos compounds. The new labels have the following values:

$$
\begin{aligned}
xc_0 &= (\tfrac{-3\sqrt{2}-2}{7},\ \tfrac{-6\sqrt{2}+10}{7})\\
xc_1 &= (\tfrac{-9\sqrt{2}-116}{136},\ \tfrac{-225\sqrt{2}+364}{136})\\
xc_2 &= (\tfrac{15\sqrt{2}-40}{23},\ \tfrac{30\sqrt{2}-34}{23})\\
xc_3 &= (\tfrac{-15\sqrt{2}+4}{31},\ \tfrac{-30\sqrt{2}+70}{31})\\
xc_4 &= (\tfrac{-81\sqrt{2}+101}{23},\ 1)\\
xc_5 &= (\tfrac{5\sqrt{2}-6}{4},\ \tfrac{\sqrt{2}+2}{4})\\
xc_6 &= (\tfrac{\sqrt{2}+4}{12},\ \tfrac{11\sqrt{2}-4}{12})\\
xc_7 &= (\tfrac{-9\sqrt{2}+71}{119},\ 1)\\
xc_8 &= (\tfrac{9\sqrt{2}+71}{119},\ 1)\\
xc_9 &= (\tfrac{7\sqrt{2}-2}{12},\ \tfrac{5\sqrt{2}+2}{12})\\
xc_{10} &= (\tfrac{8\sqrt{2}-5}{9},\ \tfrac{\sqrt{2}+5}{9})
\end{aligned}
$$

$$xc_{11} = \left(\frac{\sqrt{2}+5}{9}, \frac{8\sqrt{2}-5}{9} \right)$$
$$xc_{12} = \left(\frac{5\sqrt{2}+2}{12}, \frac{7\sqrt{2}-2}{12} \right)$$
$$xc_{13} = \left(1, \frac{9\sqrt{2}+71}{119} \right)$$
$$xc_{14} = \left(1, \frac{-9\sqrt{2}+71}{119} \right)$$
$$xc_{15} = \left(\frac{11\sqrt{2}-4}{12}, \frac{\sqrt{2}+4}{12} \right)$$
$$xc_{16} = \left(\frac{\sqrt{2}+2}{4}, \frac{5\sqrt{2}-6}{4} \right)$$
$$xc_{17} = \left(1, \frac{-81\sqrt{2}+101}{23} \right)$$
$$xc_{28} = \left(\frac{-30\sqrt{2}+70}{31}, \frac{-15\sqrt{2}+4}{31} \right)$$
$$xc_{29} = \left(\frac{30\sqrt{2}-34}{23}, \frac{15\sqrt{2}-40}{23} \right)$$
$$xc_{20} = \left(\frac{-225\sqrt{2}+364}{136}, \frac{-9\sqrt{2}-116}{136} \right)$$
$$xc_{21} = \left(\frac{-6\sqrt{2}+10}{7}, \frac{-3\sqrt{2}-2}{7} \right)$$
$$xc_{22} = \left(\frac{-219\sqrt{2}+328}{136}, \frac{-141\sqrt{2}+64}{136} \right)$$
$$xc_{23} = \left(\frac{-21\sqrt{2}+39}{71}, -1 \right)$$
$$xc_{24} = \left(\frac{-129\sqrt{2}+183}{23}, -1 \right)$$
$$xc_{25} = \left(\frac{-66\sqrt{2}+94}{31}, \frac{-33\sqrt{2}+16}{31} \right)$$
$$xc_{26} = \left(\frac{117\sqrt{2}-167}{511}, \frac{-468\sqrt{2}+157}{511} \right)$$
$$xc_{27} = \left(\frac{-145\sqrt{2}+178}{71}, \frac{-74\sqrt{2}+36}{71} \right)$$
$$xc_{28} = \left(\frac{-\sqrt{2}-4}{12}, \frac{-11\sqrt{2}+4}{12} \right)$$
$$xc_{29} = \left(\frac{-7\sqrt{2}+2}{12}, \frac{-5\sqrt{2}-2}{12} \right)$$
$$xc_{30} = \left(\frac{-8\sqrt{2}+5}{9}, \frac{-\sqrt{2}-5}{9} \right)$$
$$xc_{31} = \left(\frac{-\sqrt{2}-5}{9}, \frac{-8\sqrt{2}+5}{9} \right)$$
$$xc_{32} = \left(\frac{-5\sqrt{2}-2}{12}, \frac{-7\sqrt{2}+2}{12} \right)$$
$$xc_{33} = \left(\frac{-11\sqrt{2}+4}{12}, \frac{-\sqrt{2}-4}{12} \right)$$
$$xc_{34} = \left(\frac{-74\sqrt{2}+36}{71}, \frac{-145\sqrt{2}+178}{71} \right)$$
$$xc_{35} = \left(\frac{-468\sqrt{2}+157}{511}, \frac{117\sqrt{2}-167}{511} \right)$$
$$xc_{36} = \left(\frac{-33\sqrt{2}+16}{31}, \frac{-66\sqrt{2}+94}{31} \right)$$
$$xc_{37} = \left(-1, \frac{-129\sqrt{2}+183}{23} \right)$$
$$xc_{38} = \left(-1, \frac{-21\sqrt{2}+39}{71} \right)$$
$$xc_{39} = \left(\frac{-141\sqrt{2}+64}{136}, \frac{-219\sqrt{2}+328}{136} \right)$$

Figure 19 shows the Z-template for building the compound of 12 cubes, constisting of 3 Bakos compounds. The new labels have the following values:

$$\begin{aligned}
zc_0 &= (\tfrac{3\sqrt{2}-9}{8}, & -1) \\
zc_1 &= (\tfrac{2\sqrt{2}-7}{6}, & \tfrac{-\sqrt{2}-1}{3}) \\
zc_2 &= (\tfrac{25\sqrt{2}-87}{71}, & \tfrac{119\sqrt{2}-221}{71}) \\
zc_3 &= (\tfrac{132\sqrt{2}-647}{511}, & \tfrac{-177\sqrt{2}+43}{511}) \\
zc_4 &= (\tfrac{9\sqrt{2}-41}{31}, & \tfrac{63\sqrt{2}-101}{31}) \\
zc_5 &= (\tfrac{18\sqrt{2}-91}{68}, & \tfrac{27\sqrt{2}-43}{17}) \\
zc_6 &= (\tfrac{-39\sqrt{2}+35}{23}, & \tfrac{-15\sqrt{2}+17}{23}) \\
zc_7 &= (-1, & \tfrac{21\sqrt{2}-39}{71}) \\
zc_8 &= (-1, & \tfrac{129\sqrt{2}-183}{23}) \\
zc_9 &= (\tfrac{-468\sqrt{2}+157}{511}, & \tfrac{117\sqrt{2}-167}{511}) \\
zc_{10} &= (\tfrac{3\sqrt{2}-6}{2}, & 3\sqrt{2}-4) \\
zc_{11} &= (\tfrac{\sqrt{2}-3}{2}, & \sqrt{2}-1) \\
zc_{12} &= (\tfrac{-4\sqrt{2}+7}{9}, & \tfrac{7\sqrt{2}-1}{9}) \\
zc_{13} &= (\tfrac{-4\sqrt{2}+7}{9}, & \tfrac{7\sqrt{2}-1}{9}) \\
zc_{14} &= (\tfrac{-21\sqrt{2}+49}{31}, & \tfrac{39\sqrt{2}-29}{31}) \\
zc_{15} &= (1, & \tfrac{9\sqrt{2}+71}{119}) \\
zc_{16} &= (1, & \tfrac{-9\sqrt{2}+71}{119}) \\
zc_{17} &= (\tfrac{-107\sqrt{2}+219}{71}, & \sqrt{2}-1) \\
zc_{18} &= (\tfrac{-3\sqrt{2}+6}{2}, & 3\sqrt{2}-4) \\
zc_{19} &= (1, & \tfrac{-81\sqrt{2}+101}{23}) \\
zc_{20} &= (\tfrac{-\sqrt{2}+5}{6}, & \tfrac{-\sqrt{2}-1}{3}) \\
zc_{21} &= (\tfrac{-3\sqrt{2}+9}{8}, & -1)
\end{aligned}$$

The templates as shown in Figure 18 and Figure 19 can be used to build a model of the compound. However some faces are so small that these templates are not so practical when using cardboard or paper. For this reason the templates will be adjusted a bit.

1. The first face that will be left out is the triangle that is shown in Figure 19 and is defined by the vertices xz_2, zc_{12} and zc_{13}. Along the c_0v_4-c_0v_5 edge the triangle will connect to the same triangle. Along the other edges the triangles will cut out little triangles in the edges xc_9-xc_{12} and xc_{29}-xc_{32} in Figure 18. Removing the the little faces will then mean that these edges will not have the little notches.

2. The second face that will be left out are the triangles in Figure 18 defined by the vertices $c_{11}cx_1$, xc_{38} and xc_{39} and $c_{10}cx_3$, xc_{22} xc_{23}. As a consequence the notch in the edge c_5cx_1 xc_2 and in the edge xc_{19} c_5cx_0 of the same figure will

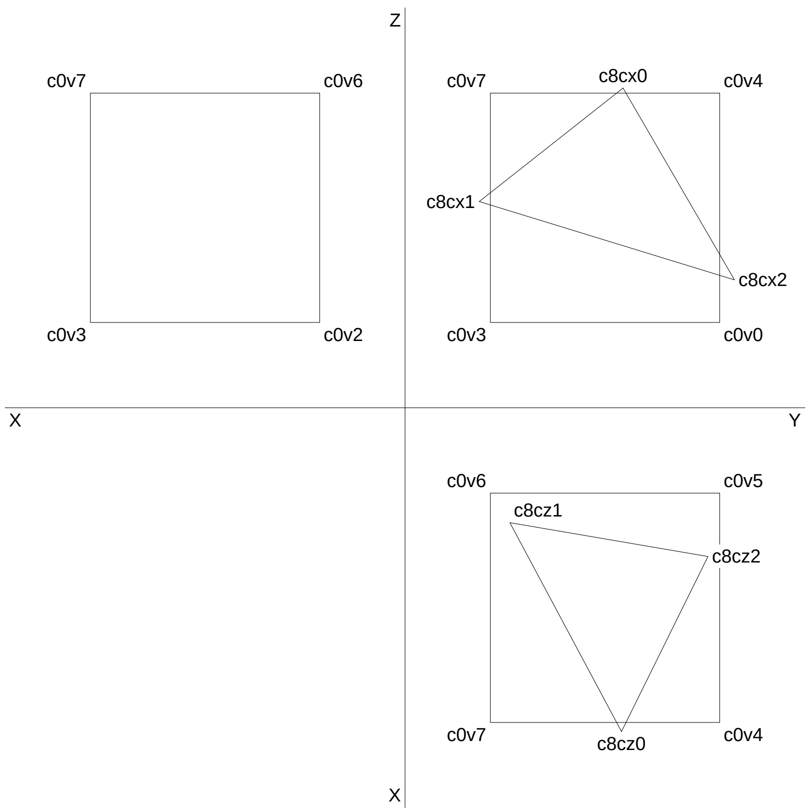

Figure 9: Intersections between The Base Cube and The Eigth Cube

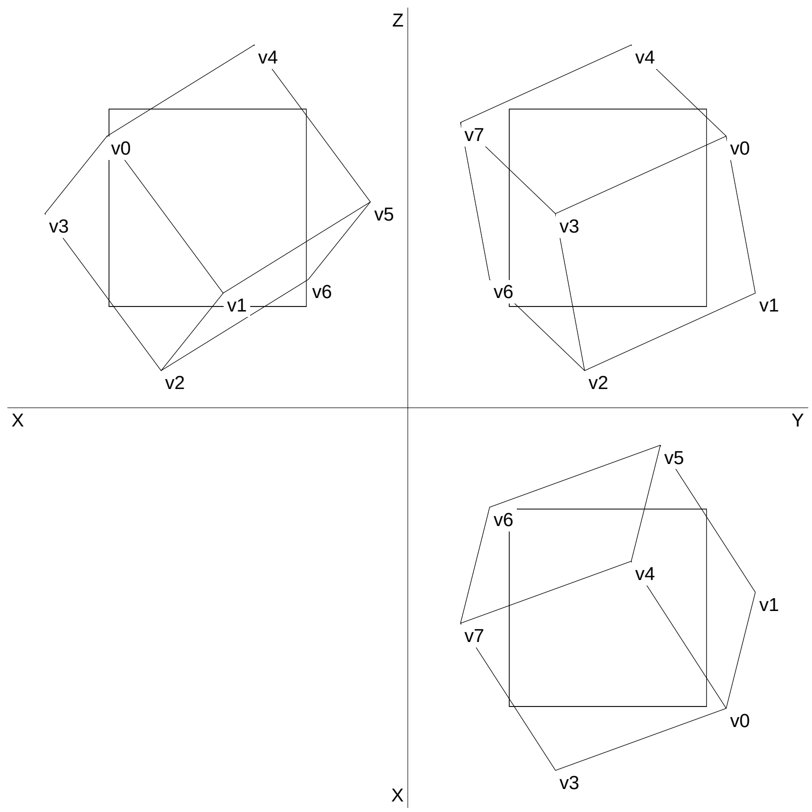

Figure 10: The Nineth Cube and Base Cube

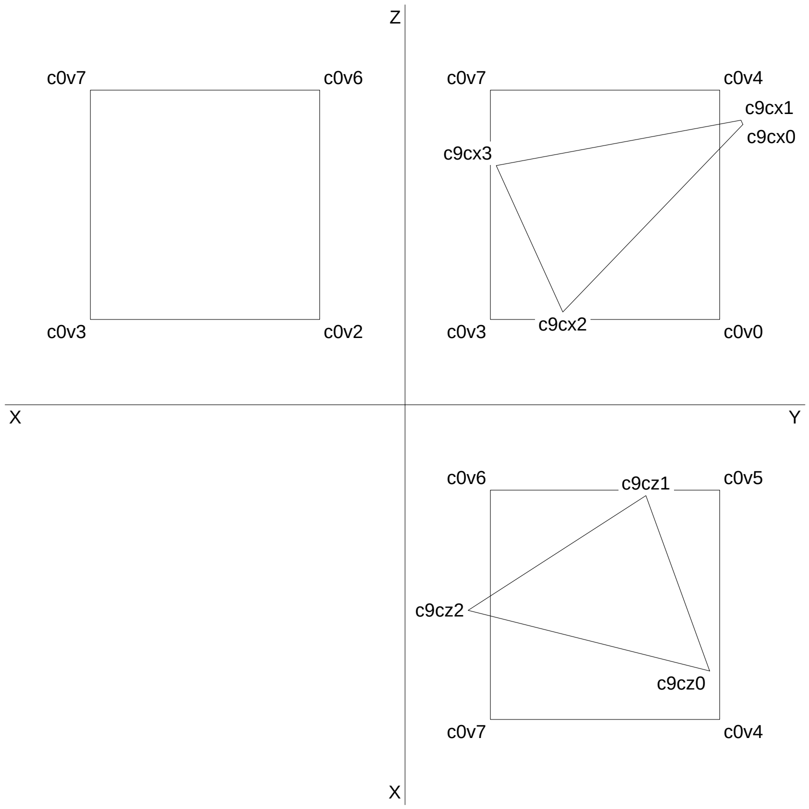

Figure 11: Intersections between The Base Cube and The Nineth Cube

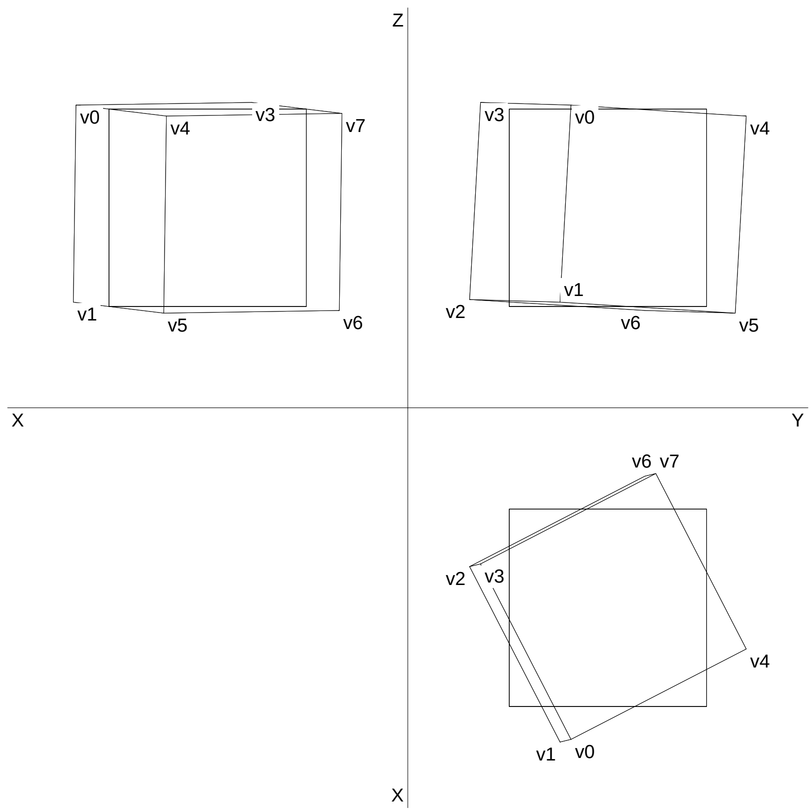

Figure 12: The Tenth Cube and Base Cube

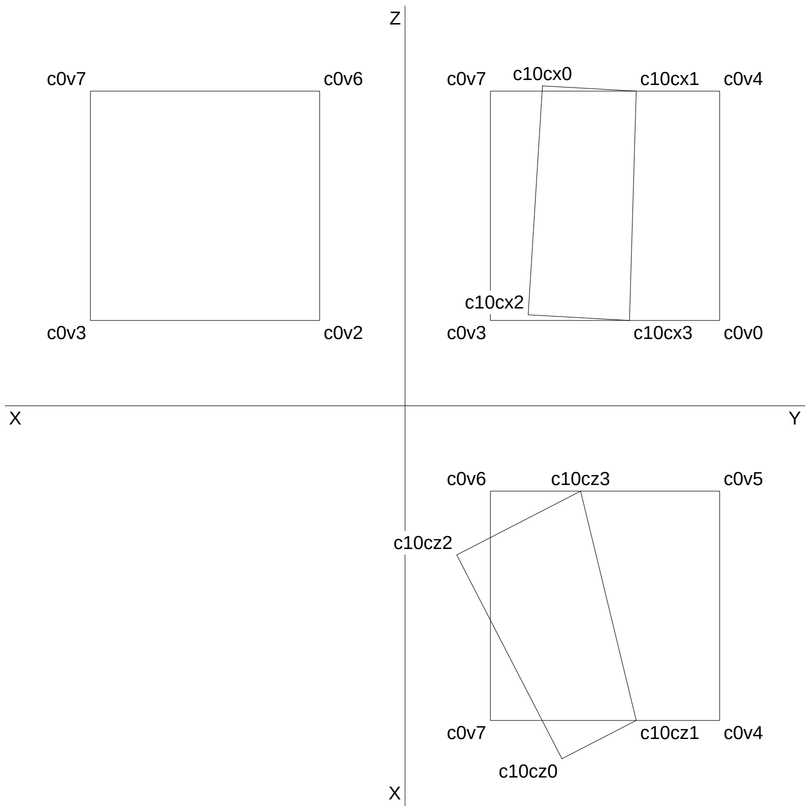

Figure 13: Intersections between The Base Cube and The Tenth Cube

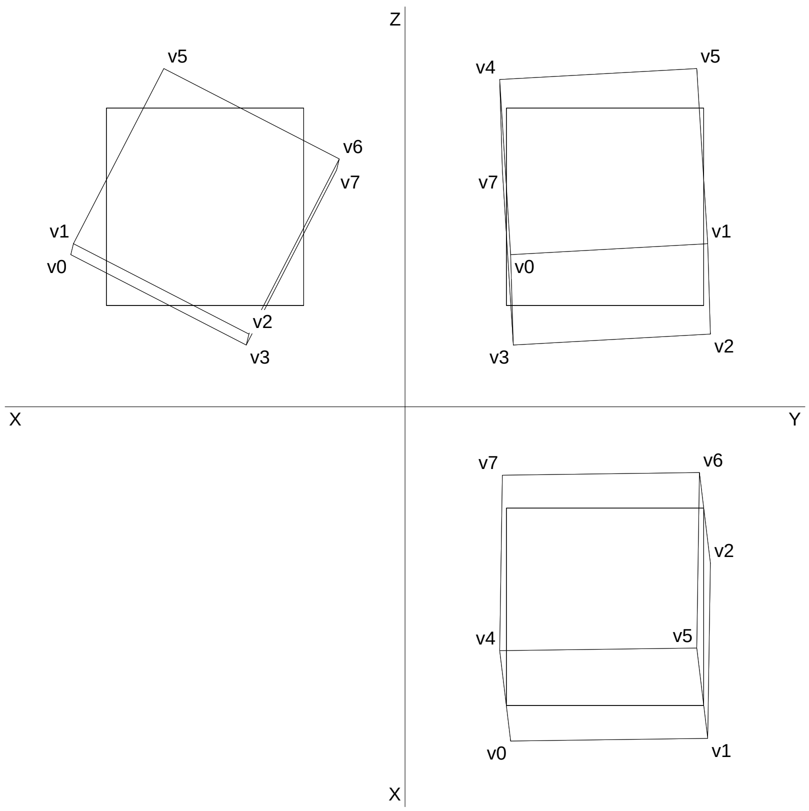

Figure 14: The Eleventh Cube and Base Cube

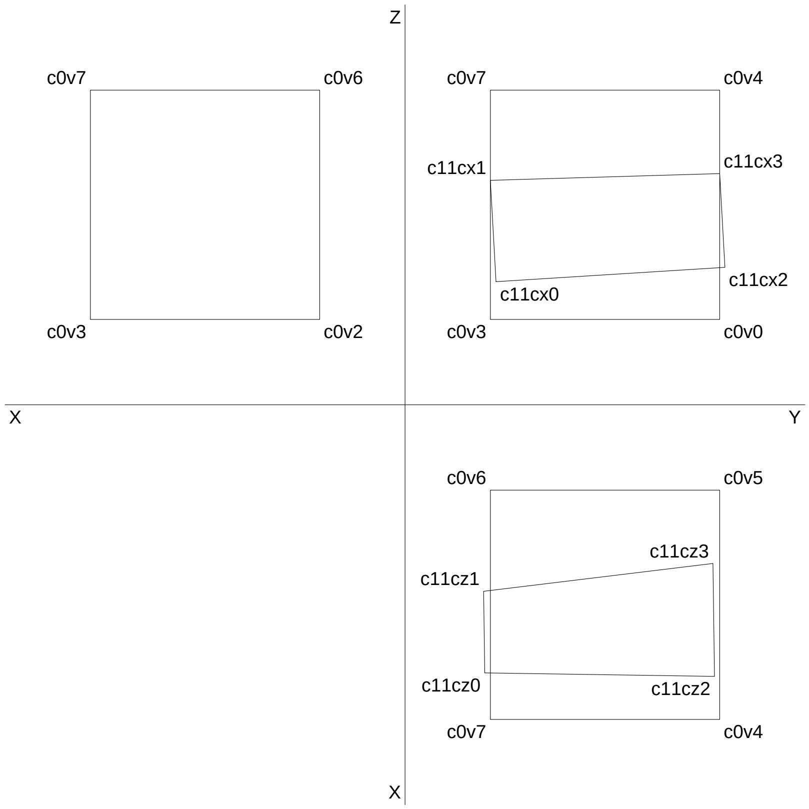

Figure 15: Intersections between The Base Cube and The Eleventh Cube

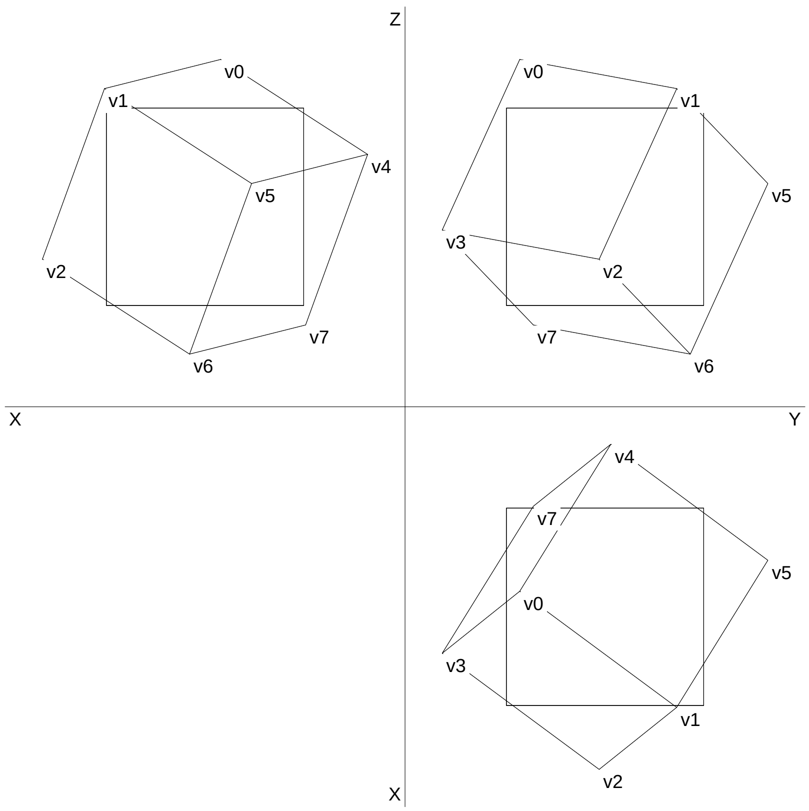

Figure 16: The Twelfth Cube and Base Cube

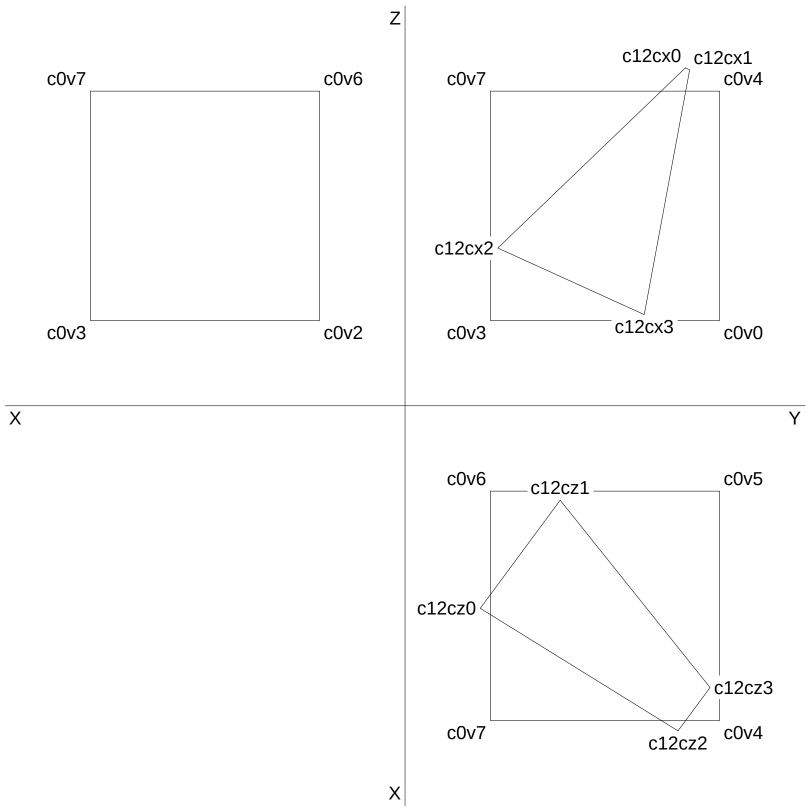

Figure 17: Intersections between The Base Cube and The Twelfth Cube

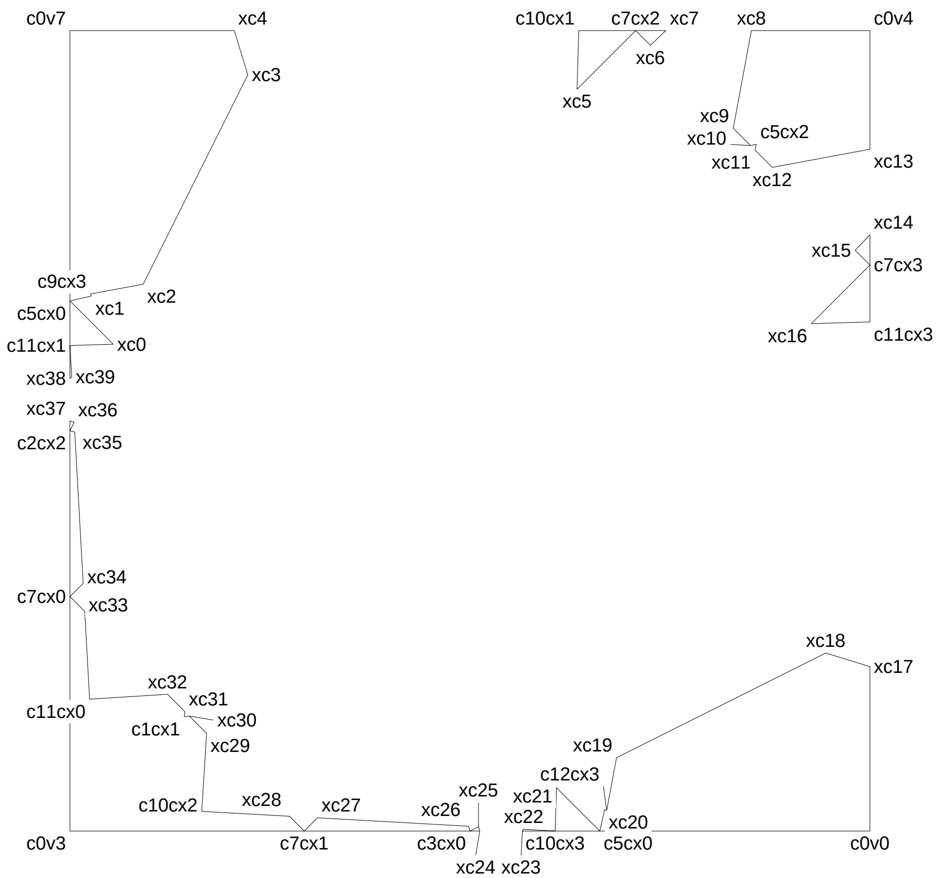

Figure 18: X-Template for building the Compound of 12 Cubes

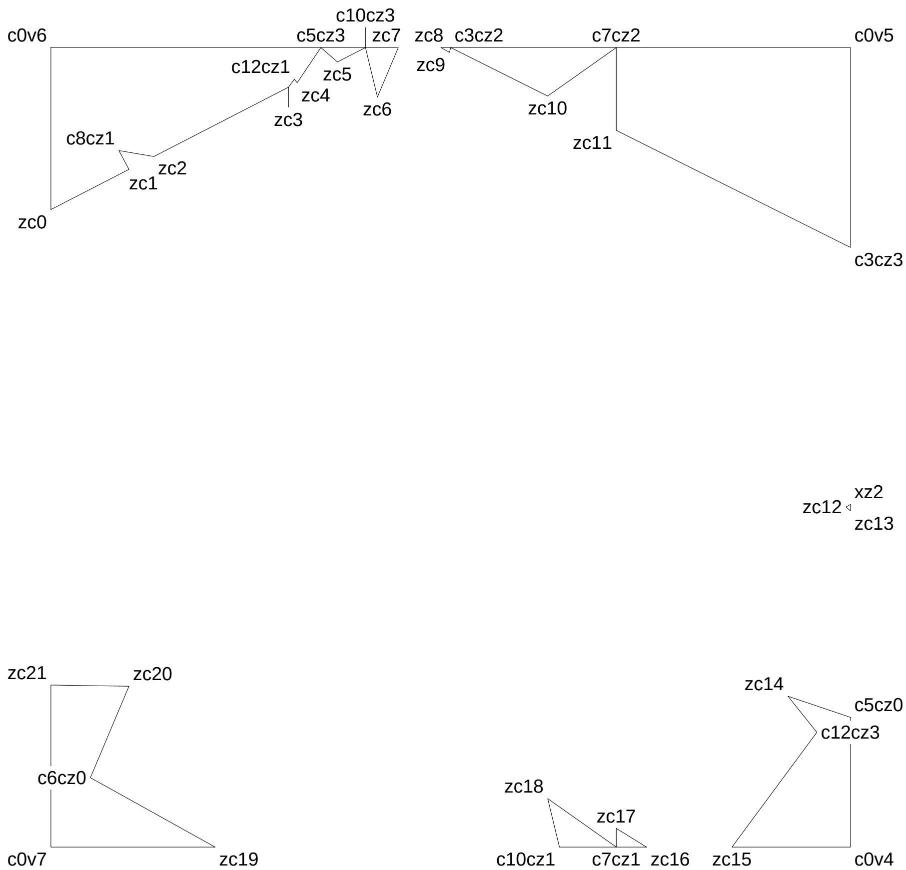

Figure 19: Z-Template for building the Compound of 12 Cubes

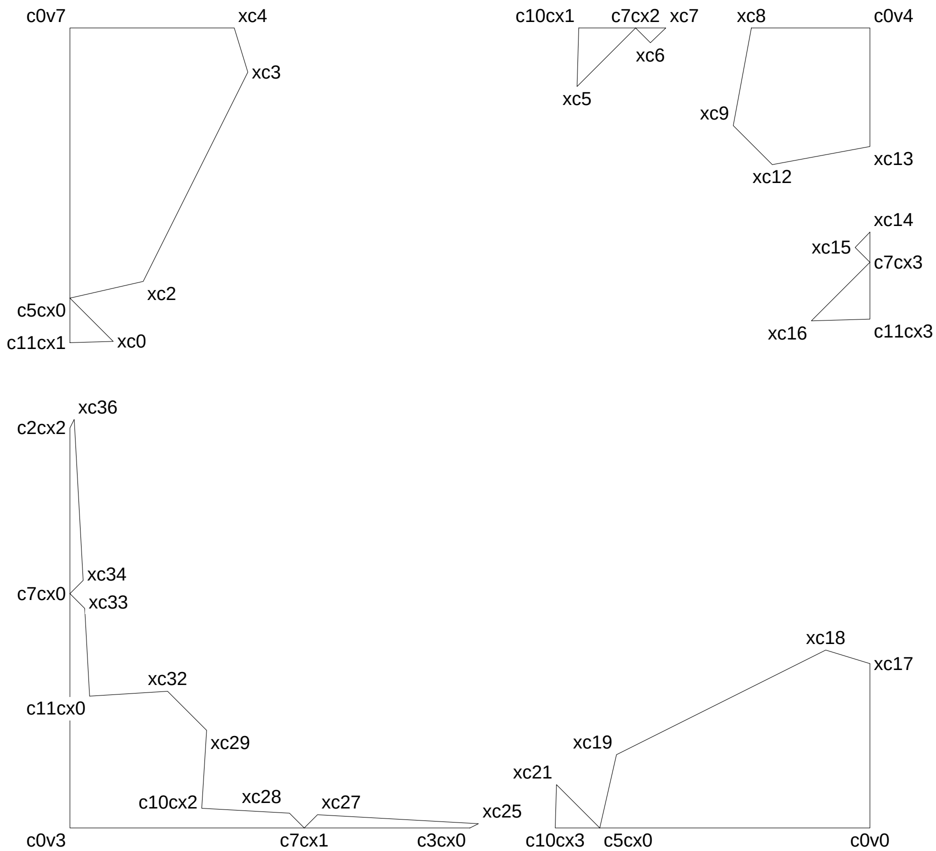

Figure 20: Adjusted X-Template for building the Compound of 12 Cubes

disappear. Also the triangle $c_{10}cz_3$, zc_6 and zc_7 in Figure 19 will be replaced by the triangle zc_5 zc_6 $c_{10}cz_3$

3. The last that will be left out is the triangles c_2cx_2 xc_{36} xc_{37} in Figure 18 and zc_7 zc_8 zc_9 and zc_{24} zc_{25} c_3cx0 in Figure 19. These triangles share an edge of the cube. The little triangles in Figure 18 will be replaced by the triangles c_2cx_2 xc_{35} xc_{36} and zc_{24} zc_{26} c_3cx0. As a consequence of leaving these little triangles out the little notch in zc_2 zc_4 in Figure 19 is removed.

With these adjustments the templates look as shown in Figure 20 and 21.

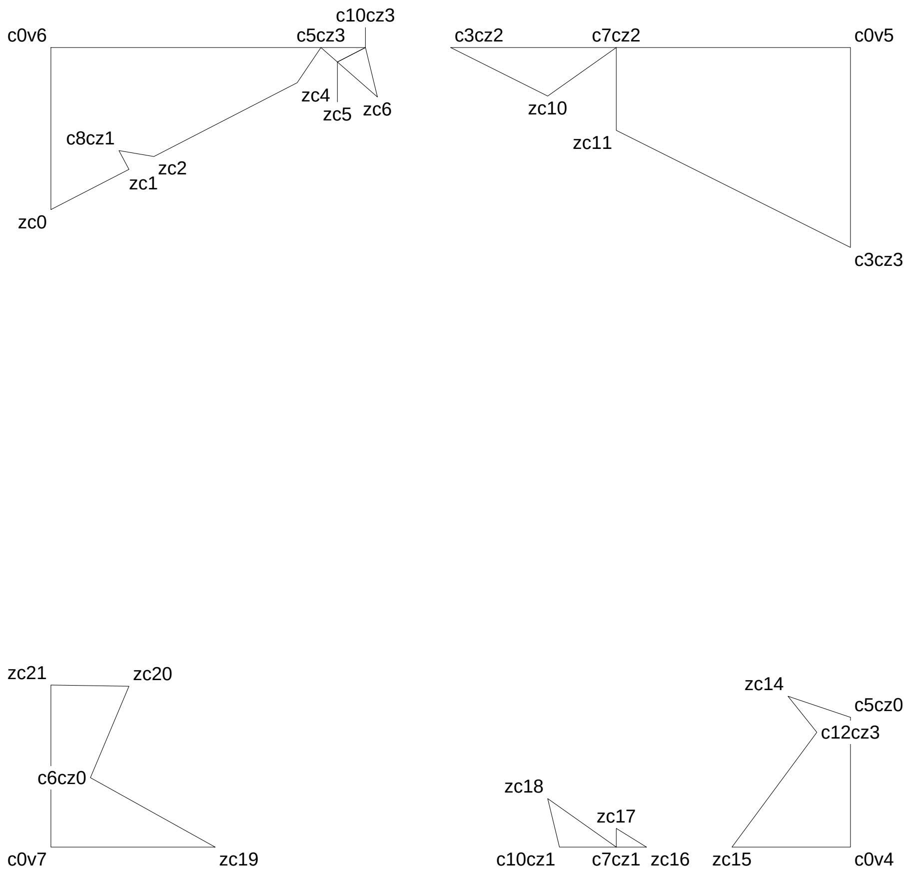

Figure 21: Adjusted Z-Template for building the Compound of 12 Cubes